Non-perturbative aspects of supersymmetric field theories

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Supersymmetry (SUSY) has not only played a pivotal role in the development of High Energy Theoretical Physics over the last decades, but it has also strongly influenced the experimental search in Particle Physics. There are many reasons to investigate it. For instance, SUSY is a central ingredient of Superstrings, the simplest and best controlled models providing a consistent and unified framework for quantum gravity and gauge interactions. It can be also used to tame ultraviolet divergences and to keep under control radiative corrections to physical observables: in supersymmetric versions of the Standard Model the Higgs mass is naturally small, since SUSY removes the quadratic divergences due to the cancellations between fermionic and bosonic loops. On the other hand supersymmetry must be broken at experimentally accessible energies: otherwise one certainly would have detected many of the additional particles it predicts.

Supersymmetric models are also of interest in itself. They are easier to investigate than non-supersymmetric ones, since they are more constrained by the higher degree of symmetry. Thus, they may serve as toy models where certain analytic results can be obtained and may serve as a qualitative guide to the behaviour of more realistic theories. In this context, over the last twenty years, there have been an enormous progress in understanding their non-perturbative dynamics and many new tools (dualities, integrability, localisation) have been developed to obtain exact results. For instance, it is possible to construct supersymmetric avatars of QCD where one can rigorously discuss the infrared theory and argue that phenomena such as confinement or chiral symmetric breaking take place. One of the main tool in extracting these results was the web of dualities linking different supersymmetric models or the same models in different regimes.

Among dualities a special role is played by is the holographic correspondence, often referred to as AdS/CFT or gauge/gravity duality. It is founded on a duality map between ordinary quantum field theories (QFTs) and higher dimensional models of gravity and strings. Remarkably, this allows suitably defined regimes, where the underlying QFT is strongly interacting, to be described by means of a classical, weakly coupled, model. Typically the dual theory contains gravity and extended objects, like fundamental strings and D-branes. In this way, hardly solvable quantum problems are mapped into easier, although non-linear, classical ones in the dual description. The original and best understood example of this kind of duality is the correspondence between $SU(N)$ gauge theory in 4 dimensions with supercharges

($\mathcal{N} = 4$) and the type IIB superstring theory on $AdS_5 \times S^5$. My PhD project will combine the above ingredients and tools to investigate different aspects of perturbative and non-perturbative features of supersymmetric and superconformal field theories.

To begin, we shall consider defect conformal field theories (dCFTs) with holographic duals: they constitute an interesting new arena for precision tests of the AdS/CFT correspondence and for the search for integrable structures. Our studies will focus on N = 4 super Yang-Mills ($\mathcal{N} = 4$ SYM) theory with a co-dimension one defect inserted at $x_3 = 0$. This setting in field theory is obtained by turning on x_3- dependent vevs for three of the scalar fields, on one side of the defect, $x_3 > 0$, while all classical fields vanish for $x_3 < 0$. Because of this peculiar Higgsing procedure, we shall have different vacua and thus gauge groups on the two sides of the defect: $SU(N)$ vs $SU(N-k)$. On the dual string theory side this corresponds to the $D5-D3$ brane system where the probe-$D5$ brane is embedded in $AdS_5 \times S^5$ so that it shares three dimensions (the defect) with the $ND3$ branes. Our first goal is to study different families of Wilson loops in this context and compute their expectation values in perturbation theory. In particular we will consider loops, analogue to the supersymmetric ones in ordinary $\mathcal{N} = 4$ super Yang-Mills. Surprisingly the output of this analysis can be directly matched with gravity computations, which are usually valid for large 't Hooft coupling λ. In fact for large k, the results can be organised in powers of the combination λ/k^2 which can be kept small independently of the value of λ. Subsequently we shall try to go beyond perturbation theory and explore the possibility of computing some of them exactly by means of localisation techniques.

Recently there have been a significant effort to extend supersymmetric dualities to case of non supersymmetric theories in $2+1$ dimensions. The starting point was the particle-vortex duality for bosonic systems established long ago in condensed matter physics, which, in its simplest version, relates the theory of a complex scalar field (the XY model) to the Abelian-Higgs model. Combining this ingredient with the so- called level-rank dualities, one can infer the existence of a web dualities relating $U(N)$ Chern-Simons (CS) theories at level k coupled to fermions, to $U(k)$ Chern- Simons theories at level N coupled to scalars. To test and enlarge our understanding these dualities, we plan to investigate the rich structure of different phases present in Chern-Simons theories coupled to matter, when studied on compact manifolds at finite temperature. The thermal partition function on S^2 is given by the Euclidean path integral of the theory on $S^2 \times S^1$ and, upon integrating out all massive modes, it reduces to compute an integral over a single unitary matrix U. The matrix U is the zero mode (on S^2) of the holonomy around the thermal circle. The full dynamics is then governed by a (generally complicated) potential function $V(U)$, whose precise form depends on the theory under study. In particular we will be interested in non-perturbative saddles in these capped unitary models since they are supposed to effectively describe the different regimes of the theory. The choice of other spatial manifolds (as the torus) and the addition of chemical potentials will be also considered. The above analysis should also lead to gain information on the behaviour of monopoles/disorder operators and other non-perturbative states through the study of the upper gap phases.