Quantum Field Theory and Topological Phases of Matter

Enrico Randellini Supervisor: Dott. Andrea Cappelli

Topological states of matter constitute a very active field of research with promising future applications in the fields of spintronics and quantum computation. Furthermore, topological phases of matter involve fundamental aspects of field theory, mathematical physics and geometry. The quantum Hall effect is the first and best know example of topological phase: a bidimensional electron system, placed in a strong magnetic field ($10T$) and at very low temperatures ($100mK$), shows a vanishing longitudinal conductivity, while the transverse, or Hall, conductivity acquires integer or rational values of e^2/h in a very precise and universal way, independently of the details of the materials.

At energies below the bulk gap, the universal behavior of the system is described by an effective action involving the Chern-Simons term. This topological gauge theory accounts for the response of the system to external electromagnetic disturbances, the quantization of the Hall conductivity, the incompressibility of the quantum Hall fluid and the fractional charge and statistics of the excitations, that are called anyons. The low-energy degrees of freedom are located at the boundary; these excitations are massless, chiral, and can be described by a $(1+1)$ dimensional conformal field theory.

In the last ten years, the list of observed topological phases of matter has been expanding; in particular, there exist the so-called topological insulators in two and three dimensions, that are realized in systems with strong spin-orbit coupling and time-reversal symmetric interactions. Also these phases of matter are characterized by an insulating bulk and massless edge modes, that are chiral and antichiral fermions in two dimensions, or a single Dirac cone at the surface of a three dimensional system.

The purpose of my thesis is to understand the stability criteria and the universal features of some topological phases of matter by the use of techniques related to the quantum field theory. In the case of the quantum Hall effect, starting by the W-infinite symmetry of the incompressible bulk, we developed a low energy effective action that, in addition to the matter vector current, allowed the introduction of a conserved spin-two current interacting with a curved metric background [1]. This construction shows the presence of a second universal quantity, called Hall viscosity. Future investigations are devoted to understand if the system can support an infinite number of universal quantities since the W-infinite symmetry allows to introduce an infinite number of independent currents of higher spin.

In the case of the two dimensional time-reversal invariant topological insulators, our studies on the partition functions of the edge excitations allowed to understand that the $\mathbb{Z}_2$ free classification is maintained to the interacting level, when the system can support non-Abelian excitations [2]. Moreover, studying the response of the system to electromagnetic and geometric variations of the background, we understood that the stability is related to the presence of a discrete $\mathbb{Z}_2$ anomaly of the ground state. Our analysis also explain what are the interactions that gap those systems not protected by the $\mathbb{Z}_2$ anomaly without breaking the time reversal symmetry [3]. We are currently studying the three dimensional case. The analysis of the partition function of the free Dirac surface fermion show that the free classification is again $\mathbb{Z}_2$. We are trying to verify that the so called BF theory is the correct effective field theory to discuss the universal features of the interacting phases.

References

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[2] A. Cappelli, E. Randellini, "Partition Functions and Stability Criteria of Topological Insulators", JHEP 12 (2013) 101.

[3] A. Cappelli, E. Randellini, "Stability of Topological Insulators with Non-Abelian Edge Excitations", J. Phys. A: Math. Theor. 48 (2015) 105404.