# Topological Phases of Matter and Bosonization

**PhD student**: Lorenzo Maffi **Supervisor**: Andrea Cappelli

In topological phases, the many body quantum states are characterized by topological invariant quantities that are indipendent of microscopic details. The strongly interacting electrons give rise to universal macroscopic phenomena that are rather unusual and fascinating. In order to be understood, these phenomena require advanced tools in quantum field theory, mathematical physics and geometry.

Whereas the Ginzburg-Landau theory provides the framework for understanding phases of matter such as ferromagnets and superfluids, topological gauge theories are used to model the universal and global properties of the new phases. Moreover dynamical and infinite-dimensional symmetries, like $2d$ conformal invariance, are used as well.

The best known example of topological phase is provided by the quantum Hall effect. A bidimensional electron system placed in a strong magnetic field $(5-10\ T)$ and at very low temperatures $(10-100\ mK)$ realizes several non-perturbative stable states, whose transverse conductivity, so-called Hall conductivity, is quantized in integer or rational values of $e^2/h$. The Hall conductivity is roboust under continuous deformations of the Hamiltonan and is a topological quantity. This roboustness is due to the presence of a gap in the bulk of the system which therefore freezes the bulk dynamic. Furthermore there arise massless fermionic escitations at the boundary, which are chiral, as magnetic field breaks the time reversal and parity symmetries. These edge excitations realize conformal field theories in two space-time dimensions. The universality of the Hall conductivity is related to the chiral anomaly in the conformal field theory at the edge.

A Chern-Simons topological theory in $(2+1)$ space-time dimensions describes the response of the system to electromagnetic disturbances, such as the Hall conductivity. For energies below the gap, this theory does not involve propagating particles, but models the degrees of freedom related to the collective behavior of the system, such as massless degrees of freedom at the boundary. In this approach, the edge excitations are bosonic, while in the microscopic description af Hall state they are fermionic. The two pictures agree thanks to the well-known bosonization map between two-dimensional field theories.

In the last ten years more topological phases of matter have been discovered, such as the topological insulators in two and three spatial dimensions. The topological states have been classified in 10 families by studying the invariance properties under the discrete trasformation $C$, $P$ and $T$.

My PhD project concerns the analysis of topological quantities and the bosonization map in the quantum Hall effect and in other states. In my Master Thesis, we have studied the universal response of a quantum Hall droplet under mechanical strains, the so-called Hall viscosity, that is related to the intrinsic angular momentum of the low-energy excitations. Studying this quantity in the conformal theory at the boundary, we understood its universal properties. My research plan involves the three-dimensional topological insulators, possesing edge excitations in $(2+1)$-dimensions. In this topological phase of matter, the boundary theory exists in both the bosonic and fermionic descriptions. I shall study the extension of the two-dimensional bosonization map in this context by using several techniques of field theory.