A Spin Squeezed Atom Interferometer

PhD Dissertation
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Light and matter-wave interference

Superposition principle of light and matter waves

\[ I \propto \left| E_1 + E_2 e^{i\phi} \right|^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \phi \]
The Mach-Zehnder atom interferometer

\[ \Phi = k_{\text{eff}} z - \omega t + \phi \quad \Delta \Phi = \Phi_u(0) - \Phi_u(T) - \Phi_1(T) + \Phi_u(2T) \]

\[ \Delta \Phi = k_{\text{eff}} g T^2 \]

Large phase resolution

Large Momentum Transfer

Long coherence times
Large momentum transfer and coherence time

Quantum superposition at the half-metre scale

Long-Lived Bloch Oscillations with Bosonic Sr Atoms and Application to Gravity Measurement at the Micrometer Scale
G. Ferrari, N. Poli, F. Sorrentino, and G. M. Tino
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(Received 21 April 2006; published 9 August 2006)

We report on the observation of Bloch oscillations on the unprecedented time scale of several seconds. The experiment is carried out with ultracold bosonic $^{89}$Sr atoms loaded into a vertical optical standing wave. The negligible atom-atom elastic cross section and zero angular momentum in the ground state makes $^{89}$Sr an almost ideal Bose gas, insensitive to typical mechanisms of decoherence due to thermalization and external stray fields. The small size of the system enables precision measurements of forces at micrometer scale. This is a challenge in physics for studies of surfaces, Casimir effects, and searches for deviations from Newtonian gravity predicted by theories beyond the standard model.

DOI: 10.1103/PhysRevLett.97.090402
PACS numbers: 03.75.—b, 04.80.—y, 32.80.—t
Approaching the shot noise limit

Total systematic uncertainty: 92 ppm
Statistical uncertainty: 116 ppm

Projection noise: Standard Quantum Limit (SQL) or shot noise

\[ |\psi\rangle = \prod_{i=1}^{N} (c_{\downarrow} |\downarrow\rangle_i + c_{\uparrow} |\uparrow\rangle_i) \]

\[ \Delta \theta_{\text{SQL}} = \frac{1}{\sqrt{N}} \]
Overcoming the Standard Quantum Limit

Heisenberg limit: \[ \Delta \theta_H = \frac{1}{N} \]

Requires correlations

Entanglement and Spin Squeezing

Spin Squeezing in Optical Cavities

Cavity feedback, one-axis twisting

Measurement-induced
Atoms and photons in a box

Resonance frequency: $\omega_0$
Decay rate: $\Gamma$

Single-atom cooperativity: $\eta = \frac{4g^2}{\kappa \Gamma} = \frac{24F}{\pi k^2 \omega^2}$

Ratio of photons scattered into the cavity mode to photons scattered into free space.

Resonance frequency: $\omega_c$
Decay rate: $\kappa$
Free space scattering

Raman scattering

- Effects:
  - Adds spin noise
  - Causes decoherence

<table>
<thead>
<tr>
<th>Initial CSS</th>
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<tbody>
<tr>
<td>$S_i$</td>
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<td>$S$</td>
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Normalized variance increase:

$$\frac{4}{3} n_{sc}$$

After one scattered photon

<table>
<thead>
<tr>
<th>Initial CSS</th>
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<tbody>
<tr>
<td>$S_i$</td>
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<td>$S$</td>
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</table>

Rayleigh scattering

- Effects:
  - Causes decoherence

Contrast loss:

$$C^2 = e^{-2n_{sc}}$$
Squeezing by one-axis twisting

One-axis twisting Hamiltonian:

$$\mathcal{H} = \hbar \chi S_z^2$$

Cavity shift due to atomic index of refraction:

$$\frac{\delta \omega_c}{\kappa/2} = \left( \frac{N_{\uparrow}}{\Delta_{\uparrow}} - \frac{N_{\downarrow}}{\Delta_{\downarrow}} \right) \frac{\eta \Gamma}{2} = \alpha S_z + \beta N$$

$$\mathcal{H}_{\text{int}} \propto c^\dagger c S_z \propto \alpha S_z^2 + \beta N S_z$$

Shearing strength:

$$Q = 2N \eta n_{\text{sc}}$$

Normalized variance:

$$\sigma^2 = \frac{2}{Q} + \frac{2Q}{3N \eta} \rightarrow \frac{4}{\sqrt{3N \eta}}$$

M.H. Schleier-Smith et al., PRA 81, 021804(R) (2010)
Collective measurements and entanglement

2-1/2 spins:
$$ S = s_1 + s_2 $$

Measurement result

Atom 1 is spin up and atom 2 is spin down

One atom is spin up and the other one is spin down

State compatible with measurement result

$$ S = 0 $$

$$ S = 1 $$

$$ \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2) $$

$$ \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2) $$

$$ |\uparrow\rangle_1 |\downarrow\rangle_2 = \frac{1}{\sqrt{2}}(|S = 0, m = 0\rangle + |S = 1, m = 0\rangle) $$

$$ m = +1 $$

$$ m = 0 $$

$$ m = -1 $$

$$ \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2) $$
Attainable measurement-induced spin squeezing (on resonance)

Normal mode splitting measured through the transmitted photon phase shift:

\[ \varphi = \frac{\pi}{2} \]

\[ \omega_c = \omega_0 \]

Maximum signal and phase sensitivity at the two transmission peaks

Normalized spin variance at the photon shot noise level:

\[ \frac{(\Delta N)^2}{N} = \left[ \frac{(1 + \frac{\Gamma}{\kappa})^2}{4N \eta n_{sc}} \right] + \frac{4}{3} \alpha n_{sc} + e^{2n_{sc}} \]

- \( n_{sc} \): number of free-space scattered photons per atom
- \( \alpha \): fraction of Raman scattered photons

Demonstrated 20 dB improvement over the SQL: Hosten et al., Nature 529, 505-508 (2016)
Comparison of resonant and dispersive detection

**On resonance:**

\[ \frac{(\Delta N)^2}{N} = \left[ \frac{(1 + \frac{\Gamma}{\kappa})^2}{4N \eta n_{sc}} + \frac{4}{3} \alpha n_{sc} \right] e^{2n_{sc}} \]

- Convenient in the bad cavity regime
- Raman scattering suppressed or strongly reduced

**Dispersive detection:**

\[ \frac{(\Delta N)^2}{N} = \left[ \frac{1}{4N \eta n_{sc}} + \frac{4}{3} \alpha n_{sc} \right] e^{2n_{sc}} \]

- Convenient in the good cavity regime
- Possible strong contribution from Raman scattering
Dispersive detection of momentum states

\[ |^{1}P_{1}\rangle \]

\[ |^{3}P_{1}, p = 2nhk + \hbar k_{R}\rangle \]

\[ |^{3}P_{1}, p = \hbar k_{R}\rangle \]

\[ |^{3}P_{0}\rangle \]

\[ |^{1}S_{0}, p = 2nhk\rangle \]

\[ |^{1}S_{0}, p = 0\rangle \]

\[ \omega_{2} - \omega_{1} = 2\Delta = 2\pi n \times 28.6 \text{ kHz} \]
Attainable squeezing of momentum states

\[
\frac{(\Delta N)^2}{N} = \left[ \frac{\mathcal{L}_a(\Delta)[1 + N\eta\mathcal{L}_a(\Delta)/2]^2}{4N\eta n_{sc}[\mathcal{L}_d(\Delta)]^2} + \frac{4}{3}\alpha n_{sc} \right] e^{2n_{sc}}
\]

Absorption and dispersion profiles

\[
\mathcal{L}_a(\Delta) = \frac{\Gamma^2}{\Gamma^2 + 4\Delta^2}, \quad \mathcal{L}_d(\Delta) = -\frac{2\Delta\Gamma}{\Gamma^2 + 4\Delta^2}
\]
Cavity setup for free space atom interferometer

- Homogeneous coupling with running waves
- Atoms free to exit the cavity volume
- Possible interferometer inside the cavity
- Bragg beams in free space or in the cavity mode

Challenge: required ultracold atoms: $< 100 \text{ nK}$
Conclusions

- Atom-cavity interaction can induce spin squeezing for atom interferometers
- Dispersive and resonant detection appear both very promising
- The unique properties of the intercombination line in strontium allows to detect momentum states
- Required large momentum transfer and homogeneous coupling i.e. ultracold atomic sample
- Possible 100-fold improvement over the standard quantum limit.