PhD Project in Theoretical Physics

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Standard statistical mechanics tells us that bulk properties of a physical system usually do not depend on boundary conditions. This may be used to make powerful predictions, some of which can be confirmed by experiments or even proved mathematically. For example, Onsager solved the 2d Ising model, and found an exact formula for the free energy using periodic boundary conditions to simplify his calculations. His result can be extended to any square lattice, since the free energy does not depend on boundary conditions. This simple but important observation is taught in any statistical mechanics course. It also motivates the study of *dimer models*, since they provide a counterexample to such intuition. In fact, the free energy of these models does depend, heavily, on boundary conditions.

The research areas of the present project are *integrable*, or *exactly solvable*, models and the *limit shape phenomenon*. This phenomenon has been first observed in some dimer models, in the so-called scaling limit, where the number of sites of the lattice is sent to infinity and the lattice spacing to zero, while keeping the size of the system constant. In such limit, under particular boundary conditions, these models may exhibit spatial phase separation, with the emergence of ordered and disordered regions, sharply separated by smooth curves, known as *arctic curves*. Correspondingly, the order parameter of the model acquires spatial dependence, with a non-trivial profile, called *limit shape*. Once the arctic curve and the limit shape have been determined, the study of their fluctuations is interesting as well; these appear to possess universal behaviours, being governed by the *Tracy-Widom distribution* and by the *conformal free boson*, respectively.

Spatial phase separation is observed in a wide variety of examples, all amenable to discrete fermionic models. In the free fermion case, the phenomenon is fully understood [**1**], while, in the interacting case, the problem is hard, with very few explicit results. Given the non-perturbative nature of this phenomenon, it is convenient to study it with exact techniques, considering a particular class of models, called *integrable*. A natural interacting and integrable generalization of dimer models on the square lattice is the *six-vertex model*. It presents spatial phase separation for a wide choice of fixed boundary conditions. The interaction parameter is given by the anisotropy Δ. In particular, for $\Delta = 0$ we retrieve the above mentioned dimer model (which maps to free fermions).

In this context, the project of my PhD has two main goals:

i) The study of limit shape phenomena for generic Δ. The problem of the determination of the arctic curves has been solved [**2**], but many open questions remain, ranging from the determination of the limit shapes to the characterization of their fluctuations. In particular, a challenging question concerns the effect of a non-vanishing value of Δ on the universal behaviours mentioned above [**3**].

ii) The study of non-equilibrium evolution in quantum spin chains. Indeed, the transfer matrix of the six-vertex model can be viewed as the evolution operator of the XXZ quantum spin chain in discrete imaginary times. In this framework, the results for the six-vertex model, suitably translated to continuous real times, can be used to study phase separation and spin transport phenomena in the quantum quench of the XXZ chain [**3**, **4**].

References

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