PhD Diessertation

Firenze 16 · 02 · 2017

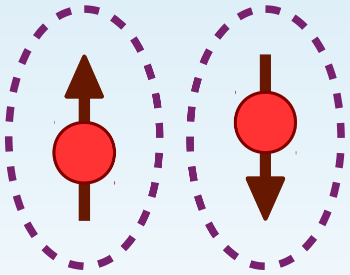
Witnessing entanglement for quantum interferometry

Marco Gabbrielli

Quantum entanglement

$$|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$$

separable



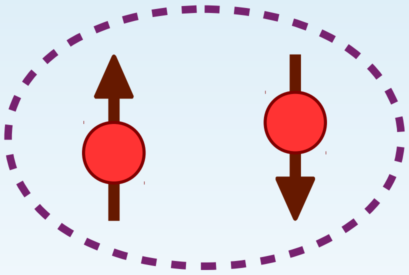
Quantum entanglement

$$|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$$

separable

$$|\psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

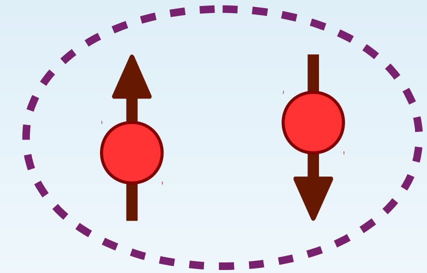
entangled



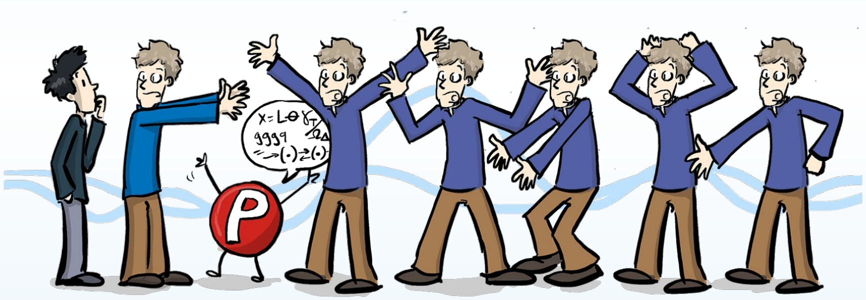
Quantum entanglement

$$|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$$ ***separable***

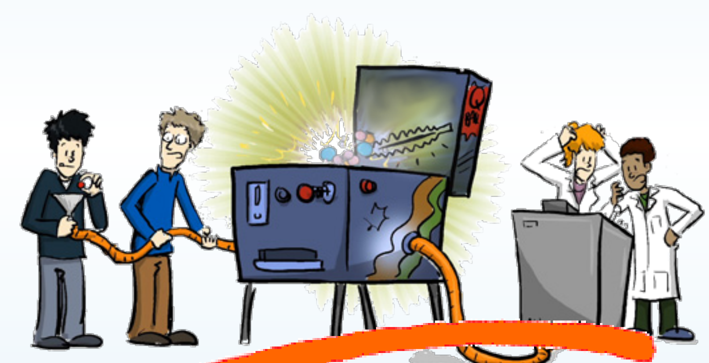
$$|\psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$ ***entangled***



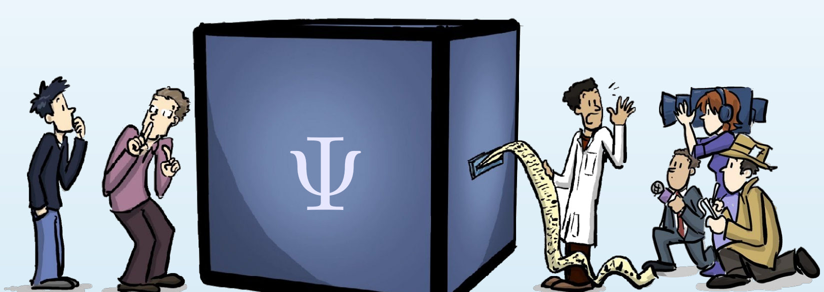
- **Applications**



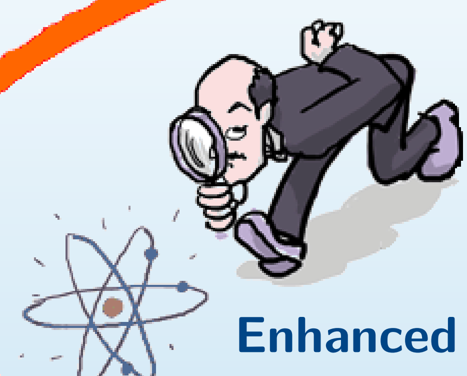
Fundations of Quantum Mechanics



Secure communication



Faster computing



Enhanced metrology

Quantum entanglement

$$|\psi\rangle = |\uparrow\rangle|\downarrow\rangle$$

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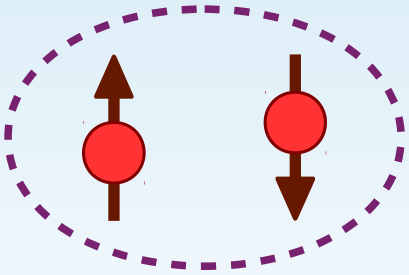
$$|\psi\rangle = \frac{|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

entangled



- **Quantifying entanglement**

$N = 2$

Bipartite entanglement measures:

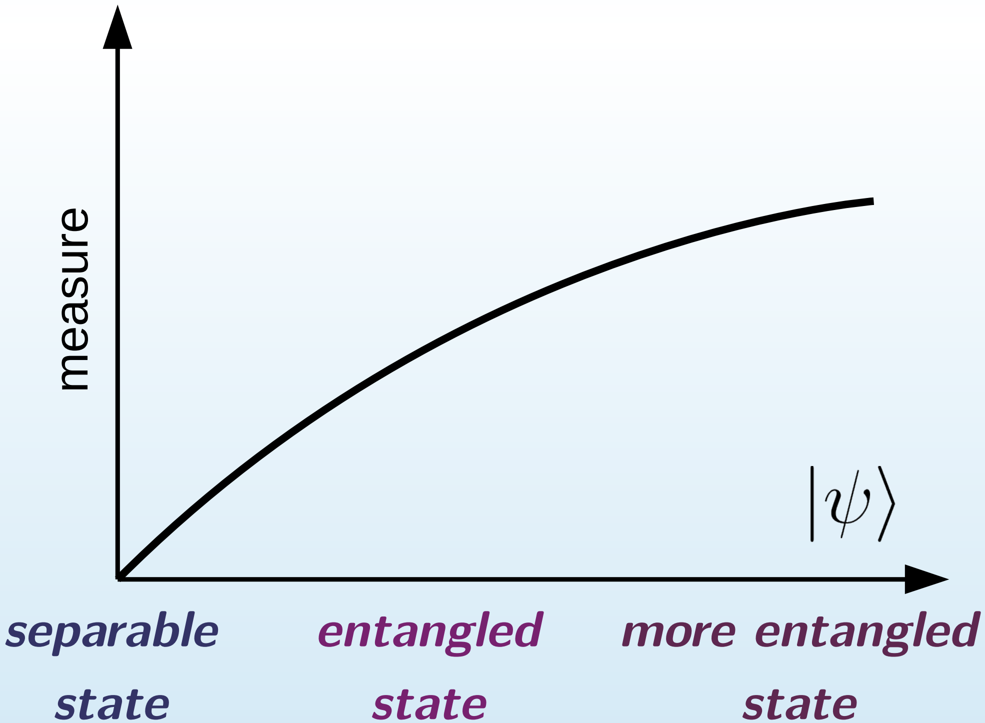
entropy

concurrence

logarithmic negativity

...

and for $N \gg 2$?



Quantum Fisher information

- **Ingredients**

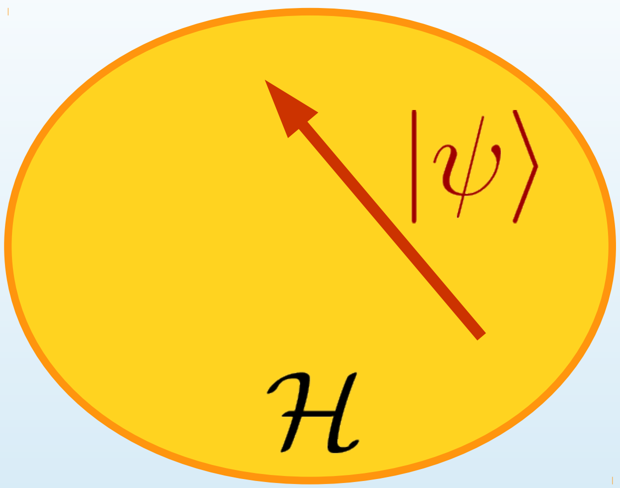
Many-body physical system $|\psi\rangle$

Unitary local transformation

Transformed state $|\psi\rangle' = U|\psi\rangle$

$$\hat{U} = e^{-i\theta\hat{H}}$$

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \cdots + \hat{H}_N$$



Quantum Fisher information

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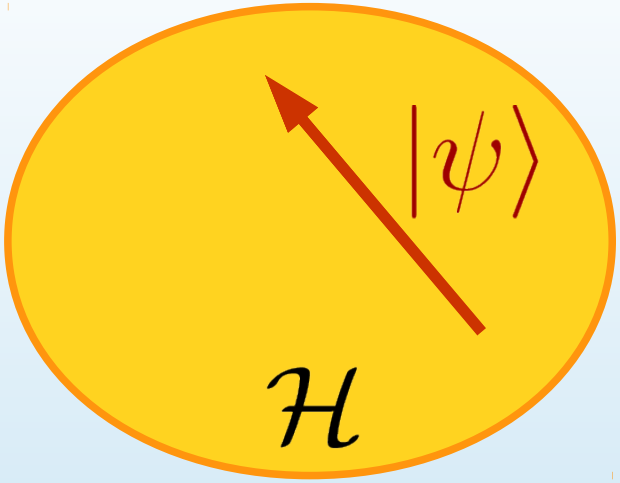
- **Recipe**

$$\boxed{F_Q[|\psi\rangle, \hat{H}] = 4(\Delta\hat{H})^2}$$



Quantum Fisher information

► **Ingredients**

Many-body physical system $|\psi\rangle$

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► **Recipe**

$$\boxed{F_Q[|\psi\rangle, \hat{H}] = 4(\Delta\hat{H})^2}$$

► **Witnessing entanglement**

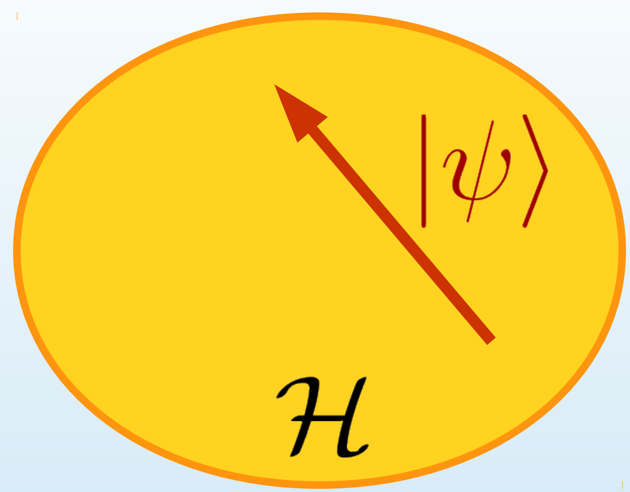
- If $|\psi\rangle$ separable $\Longrightarrow$ $F \leqslant N \quad \forall\, H$

$$|\psi\rangle = |\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle \qquad F_Q \leq 4$$

- If $\exists\, H$ such that $F > N$ $\Longrightarrow$ $|\psi\rangle$ entangled

$$|\psi\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle \qquad F_Q > 4$$

- If $\exists\, H$ such that $F > N$ $\cancel{\Longleftarrow}$ $|\psi\rangle$ entangled



Quantum Fisher information

- **Ingredients**

Many-body physical system $|\psi\rangle$

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- **Recipe**

$$\boxed{F_Q[|\psi\rangle, \hat{H}] = 4(\Delta\hat{H})^2}$$

$$\boxed{\sqrt{F_Q} \propto \frac{\mathrm{d}\mathcal{D}}{\mathrm{d}\theta}}$$

- **Witnessing entanglement**

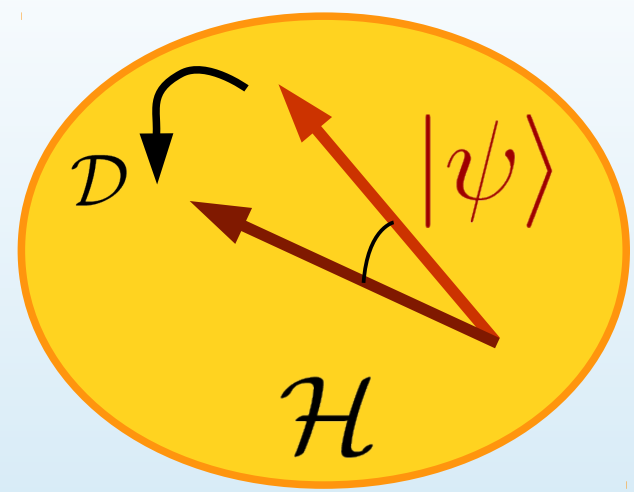
• If $|\psi\rangle$ separable $\Longrightarrow$ $F \preccurlyeq N \quad \forall\, H$

$$|\psi\rangle = |\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle \qquad F_Q \leq 4$$

• If $\exists\, H$ such that $F \succ N \Longrightarrow |\psi\rangle$ entangled

$$|\psi\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle \qquad F_Q > 4$$

• If $\exists\, H$ such that $F \succ N$ $\cancel{\Longleftarrow}$ $|\psi\rangle$ entangled



Quantum Fisher information

- **Ingredients**

Many-body physical system $|\psi\rangle$

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Transformed state $|\psi\rangle' = U|\psi\rangle$

$$\hat{U} = e^{-i\theta\hat{H}}$$

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- **Witnessing entanglement**

- If $|\psi\rangle$ separable $\Rightarrow$ $F \leqslant N \;\; \forall\, H$

$$|\psi\rangle = |\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle \qquad F_Q \leq 4$$

- If $\exists\, H$ such that $F > N$ $\Longrightarrow$ $|\psi\rangle$ entangled

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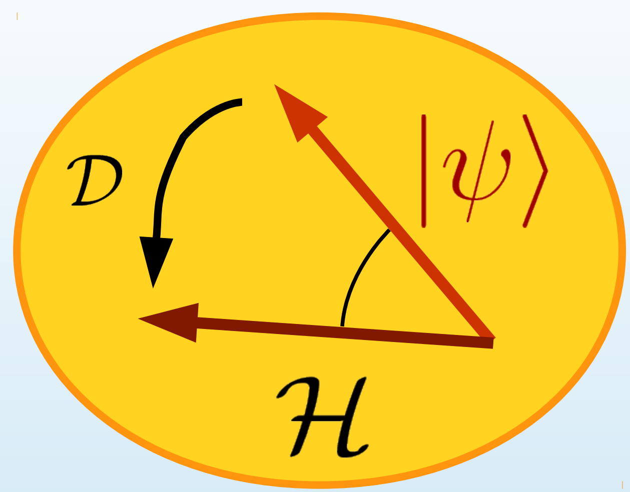

Quantum Fisher information

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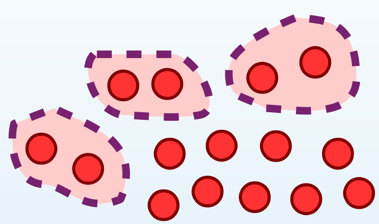
- **Recipe**

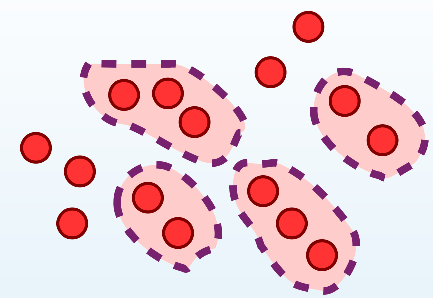
$$\boxed{F_Q[|\psi\rangle, \hat{H}] = 4(\Delta\hat{H})^2}$$

- **Witnessing entanglement *depth***

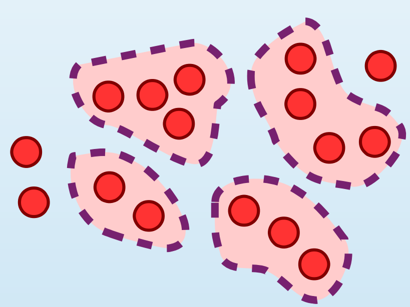
$F \succ N \Longrightarrow$



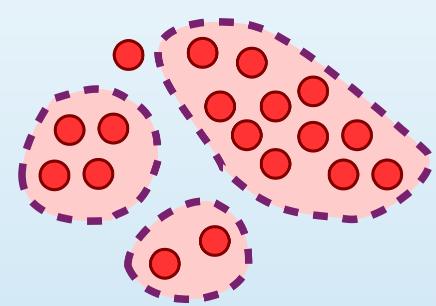
$F \succ 2\,N \Longrightarrow$



$F \succ 3\,N \Longrightarrow$



$F \succ 10\,N \Longrightarrow$



Quantum interferometry

PRL **106,** 080801 (2011) PHYSICAL REVIEW LETTERS week ending 25 FEBRUARY 2011

New Determination of the Fine Structure Constant and Test of the Quantum Electrodynamics

Rym Bouchendira,[1] Pierre Cladé,[1] Saïda Guellati-Khélifa,[2] François Nez,[1] and François Biraben[1]

LETTER

doi:10.1038/nature13433

Precision measurement of the Newtonian gravitational constant using cold atoms

G. Rosi[1], F. Sorrentino[1], L. Cacciapuoti[2], M. Prevedelli[3] & G. M. Tino[1]

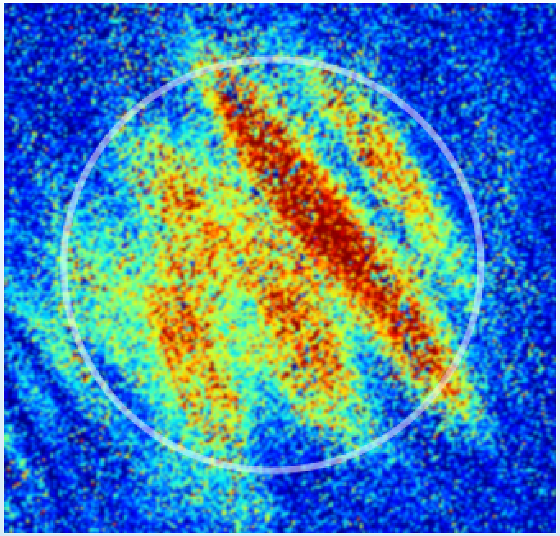
Selected for a Viewpoint in *Physics*
PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

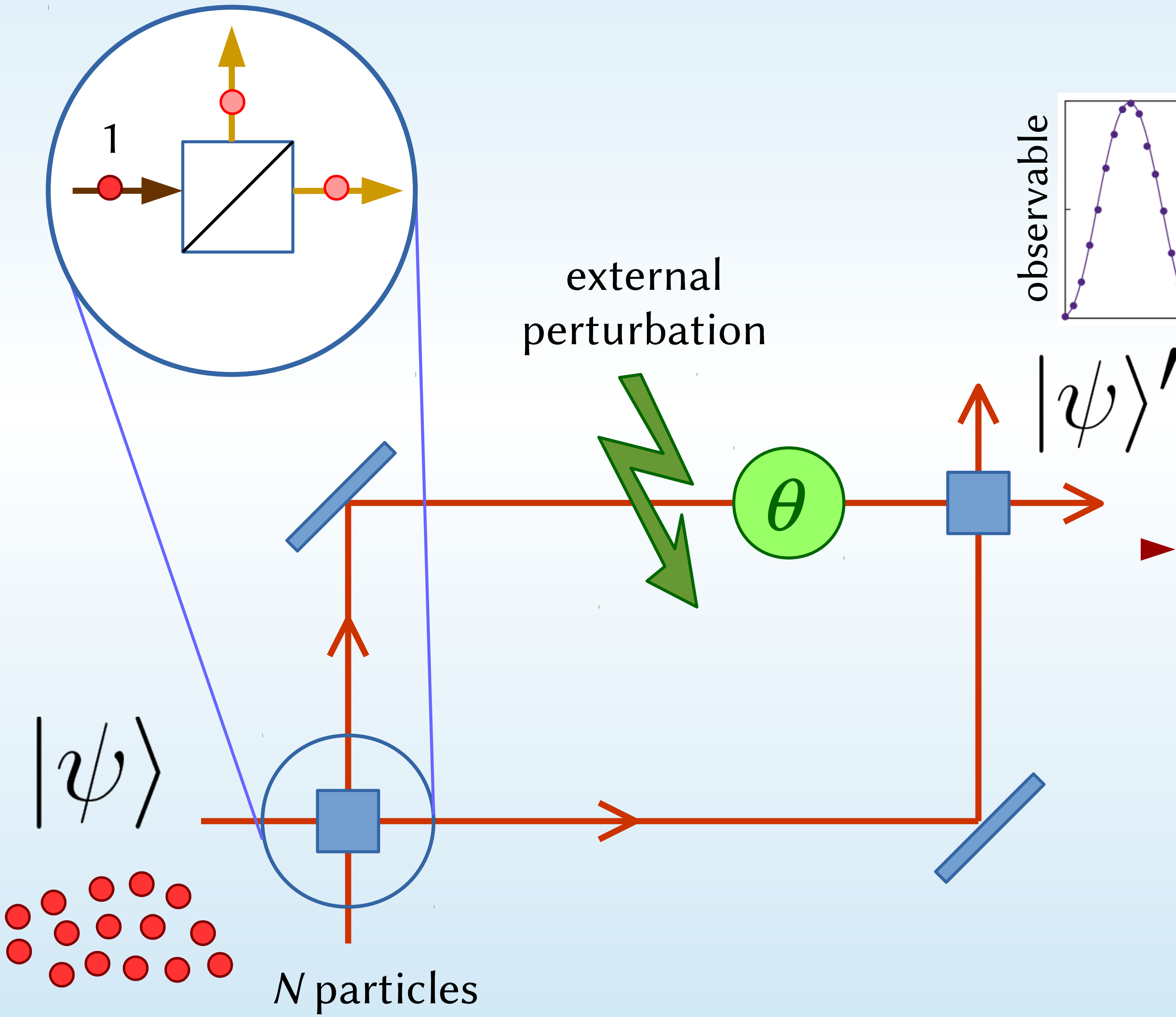
Optical interferometer (LIGO)

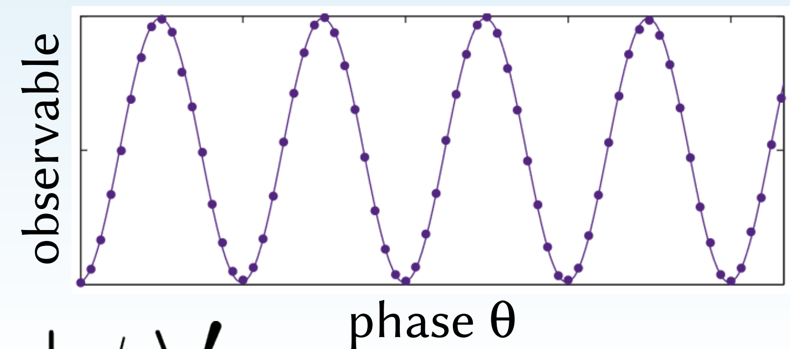


Atom interference (LENS)

Quantum *local* interferometry

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \cdots + \hat{H}_N$$





- **Phase sensitivity**

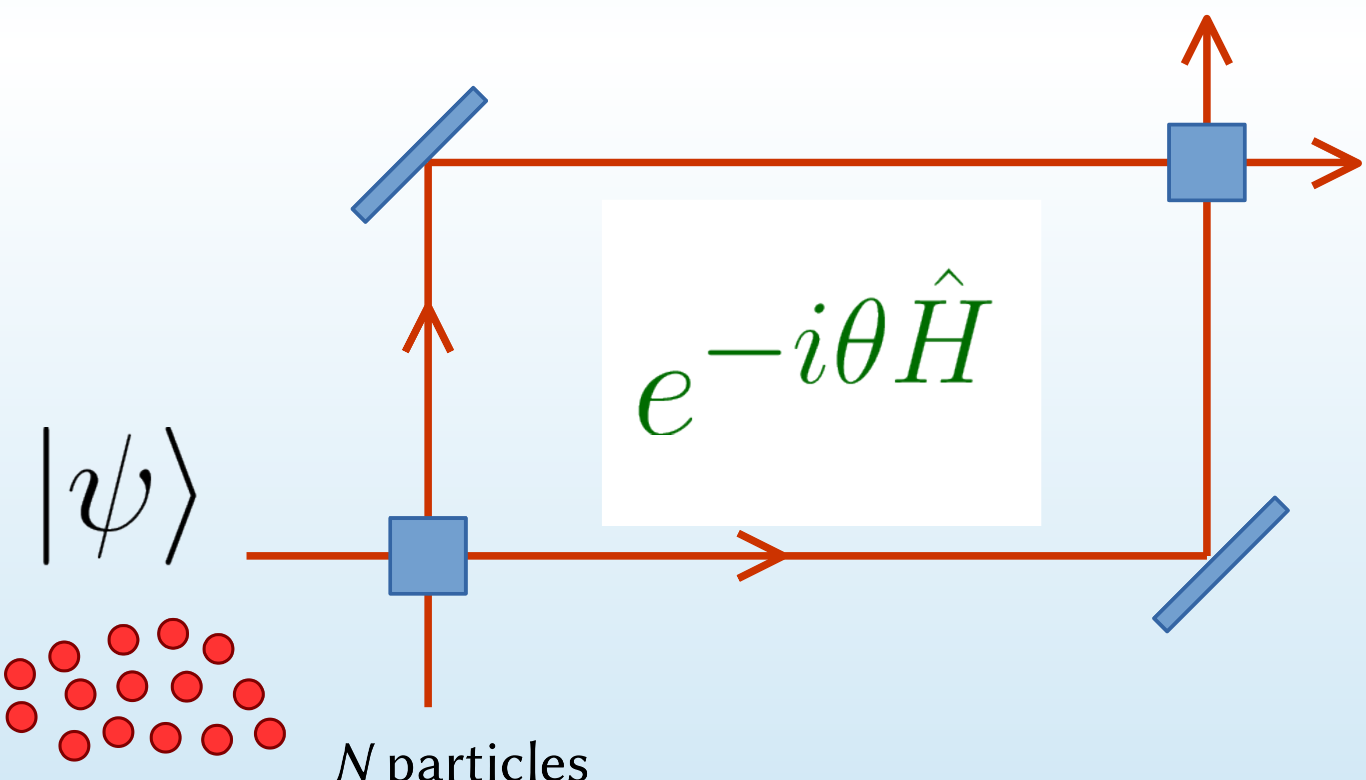
$$\boxed{\Delta\theta \geq \frac{1}{\sqrt{F_Q}}}$$

$$\Leftrightarrow$$

$$\Delta\theta\, \Delta\hat{H} \geq 1$$

Quantum *local* interferometry

- $F \leqslant N \quad \Longrightarrow \quad \Delta\theta_{min} \geq \frac{1}{\sqrt{N}}$ ***shot noise***



► **Phase sensitivity**

$$\boxed{\Delta\theta \geq \frac{1}{\sqrt{F_Q}}}$$

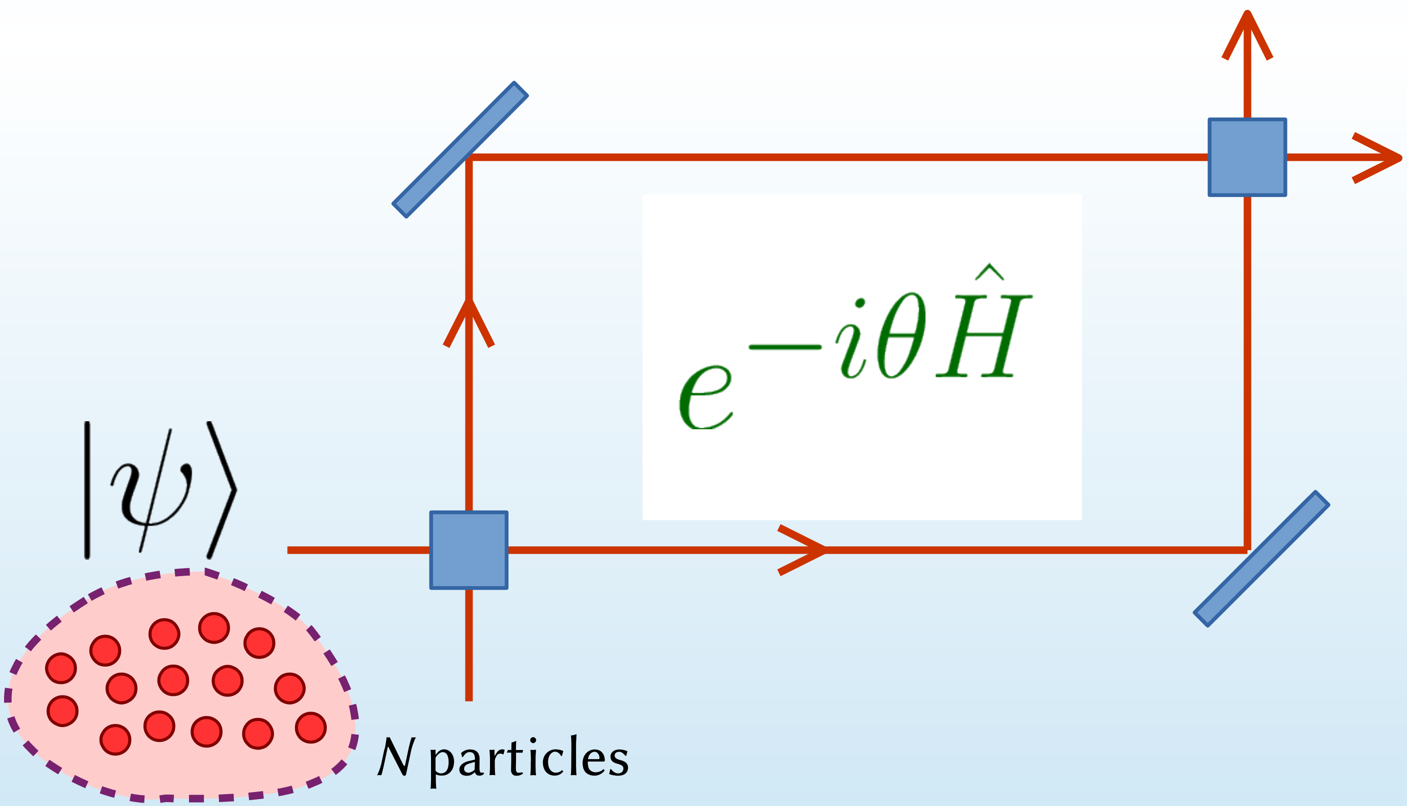
$$\Longleftrightarrow$$

$$\Delta\theta\,\Delta\hat{H} \geq 1$$

Quantum *local* interferometry

- $F \leqslant N \quad \Longrightarrow \quad \Delta\theta_{min} \geq \dfrac{1}{\sqrt{N}}$ ***shot noise***
- $F > N \quad \Longleftrightarrow \quad \Delta\theta_{min} < \dfrac{1}{\sqrt{N}}$ ***sub shot noise***

Pezzè & Smerzi, PRL (2009)



► **Phase sensitivity**

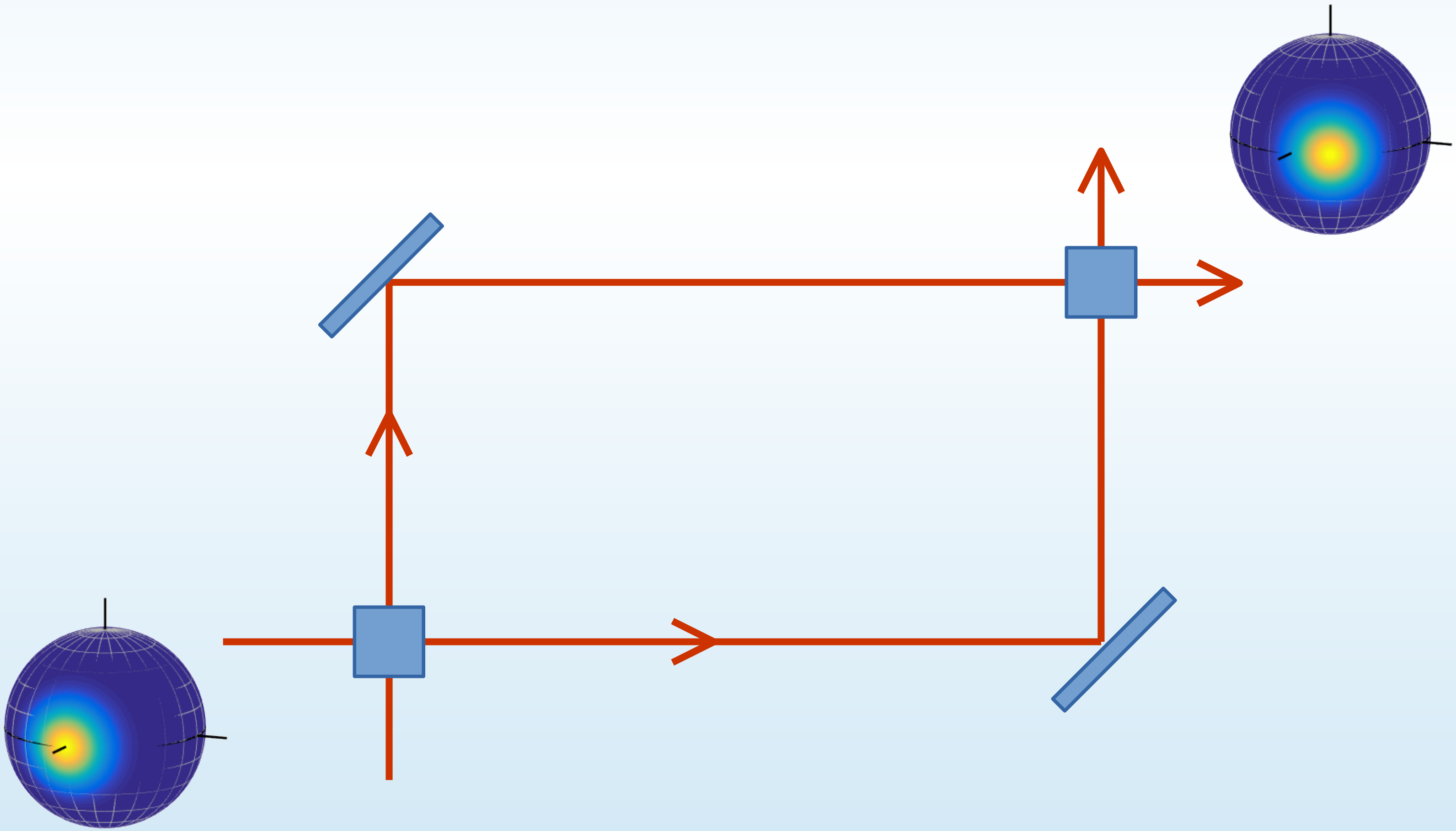
$$\boxed{\Delta\theta \geq \frac{1}{\sqrt{F_Q}}}$$

$$\Longleftrightarrow$$

$$\Delta\theta \, \Delta\hat{H} \geq 1$$

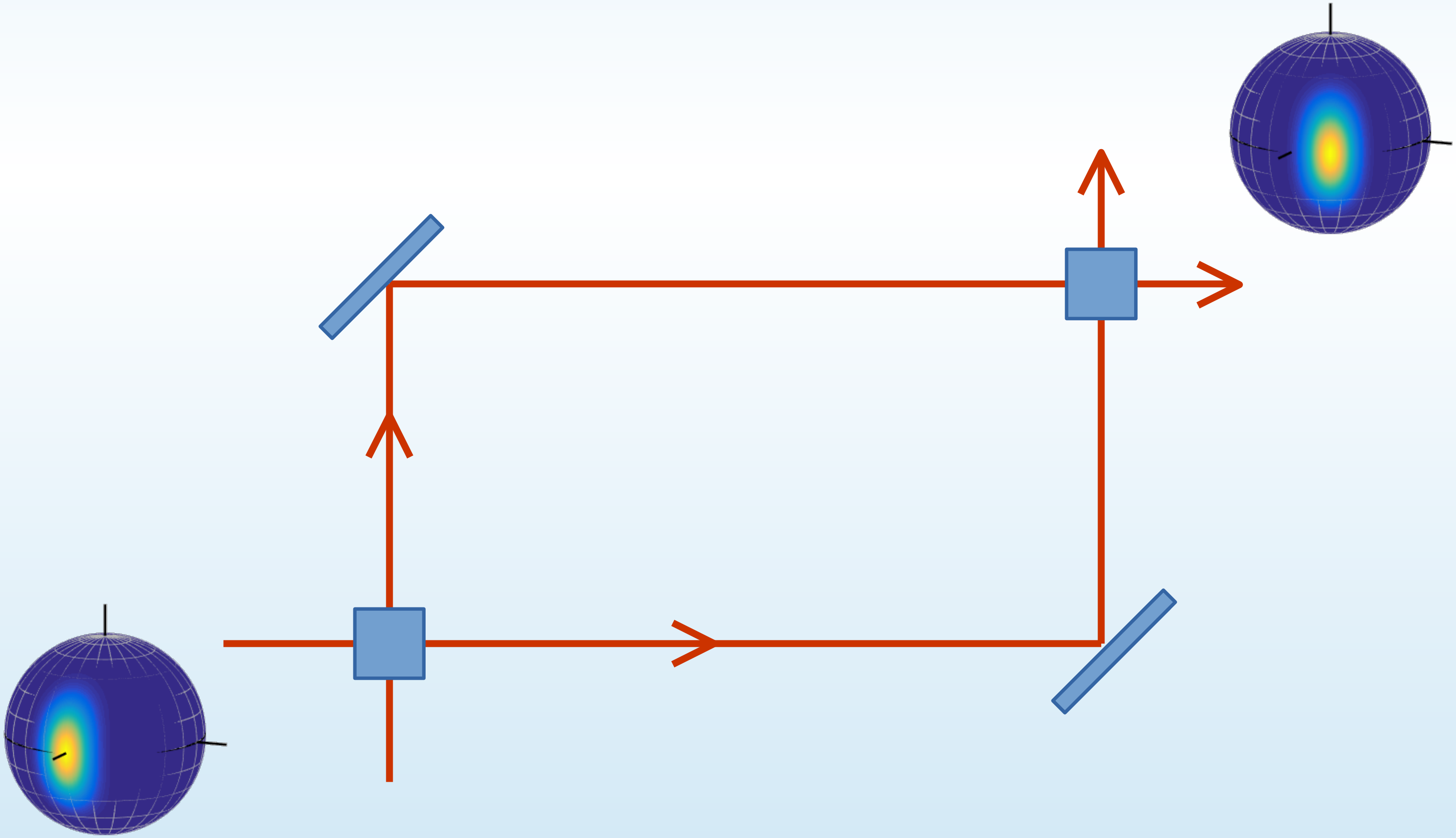
Quantum *local* interferometry

separable state

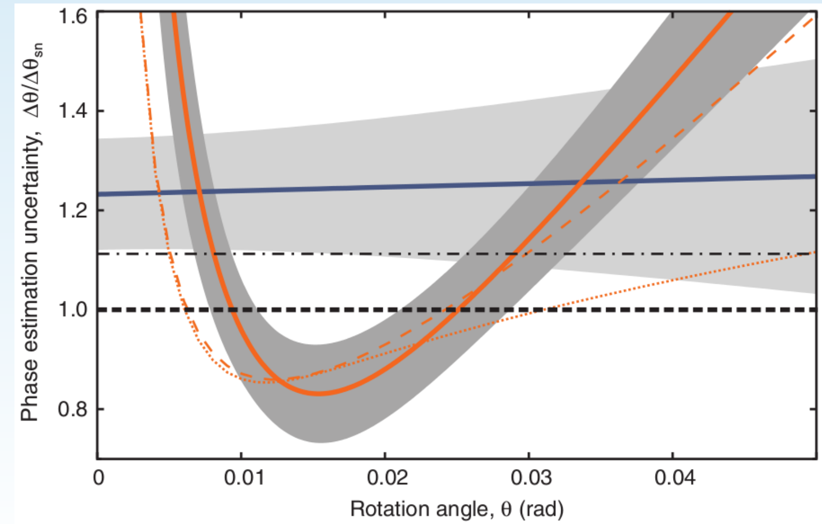


Quantum *local* interferometry

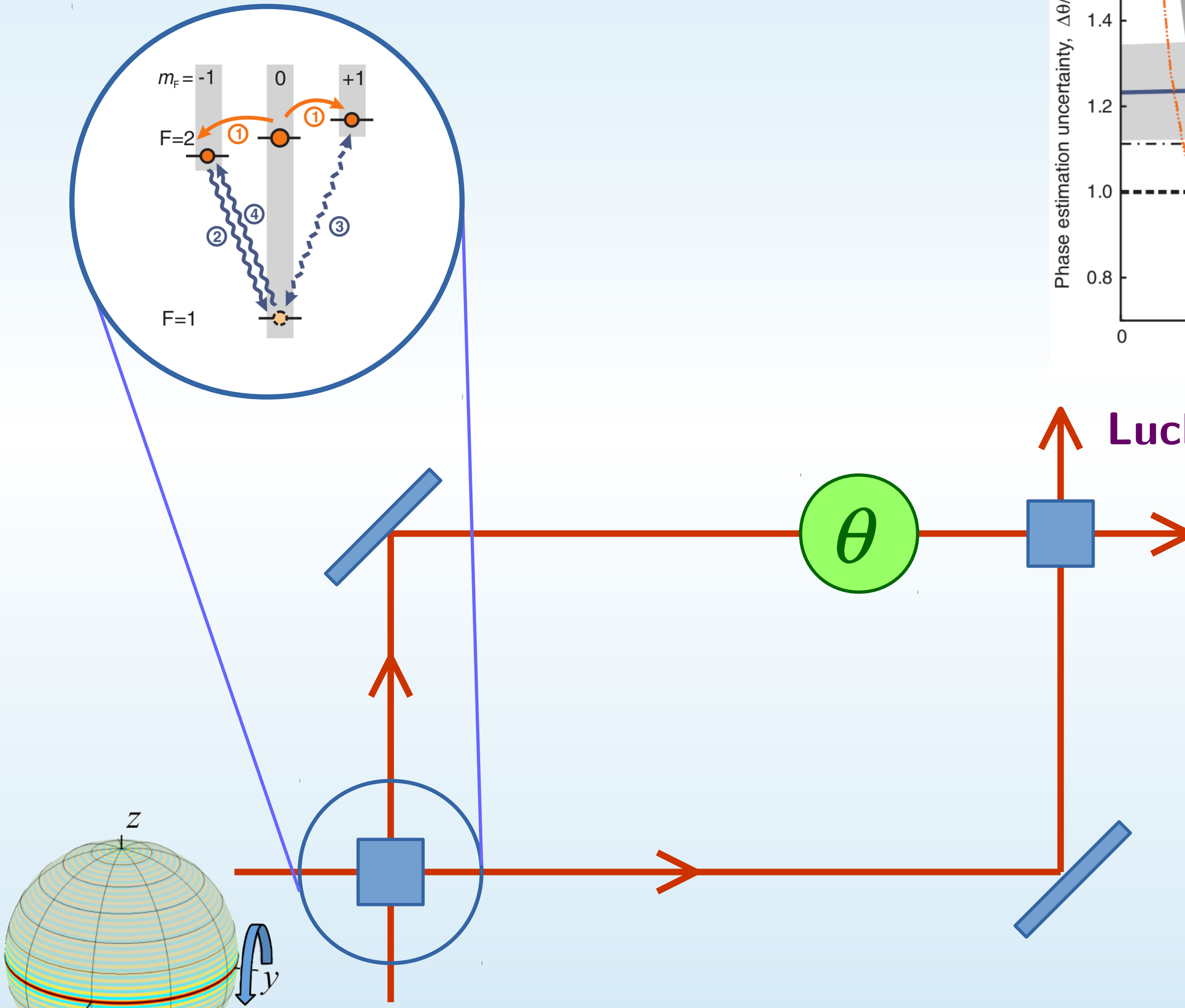
entagled states



Quantum *local* interferometry: example



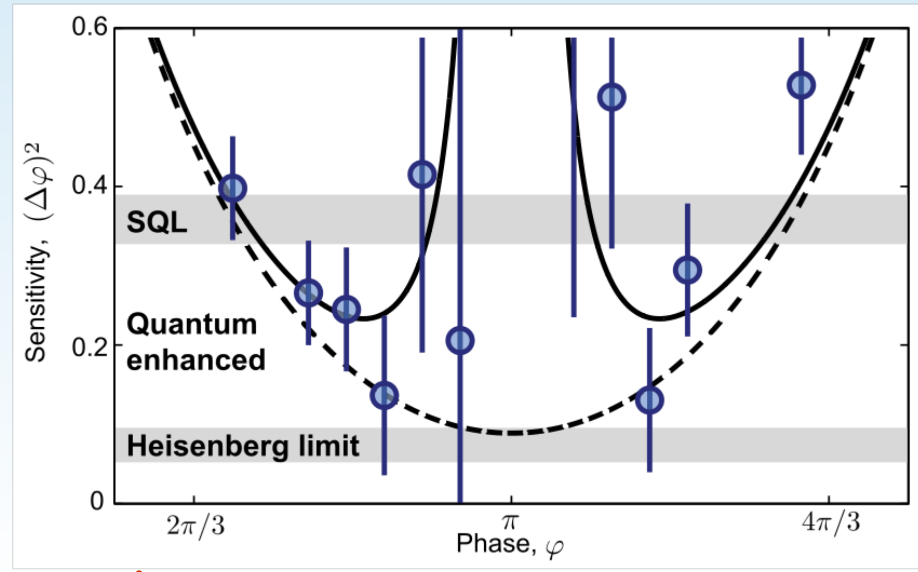
Lucke *et al*, SCIENCE (2011)



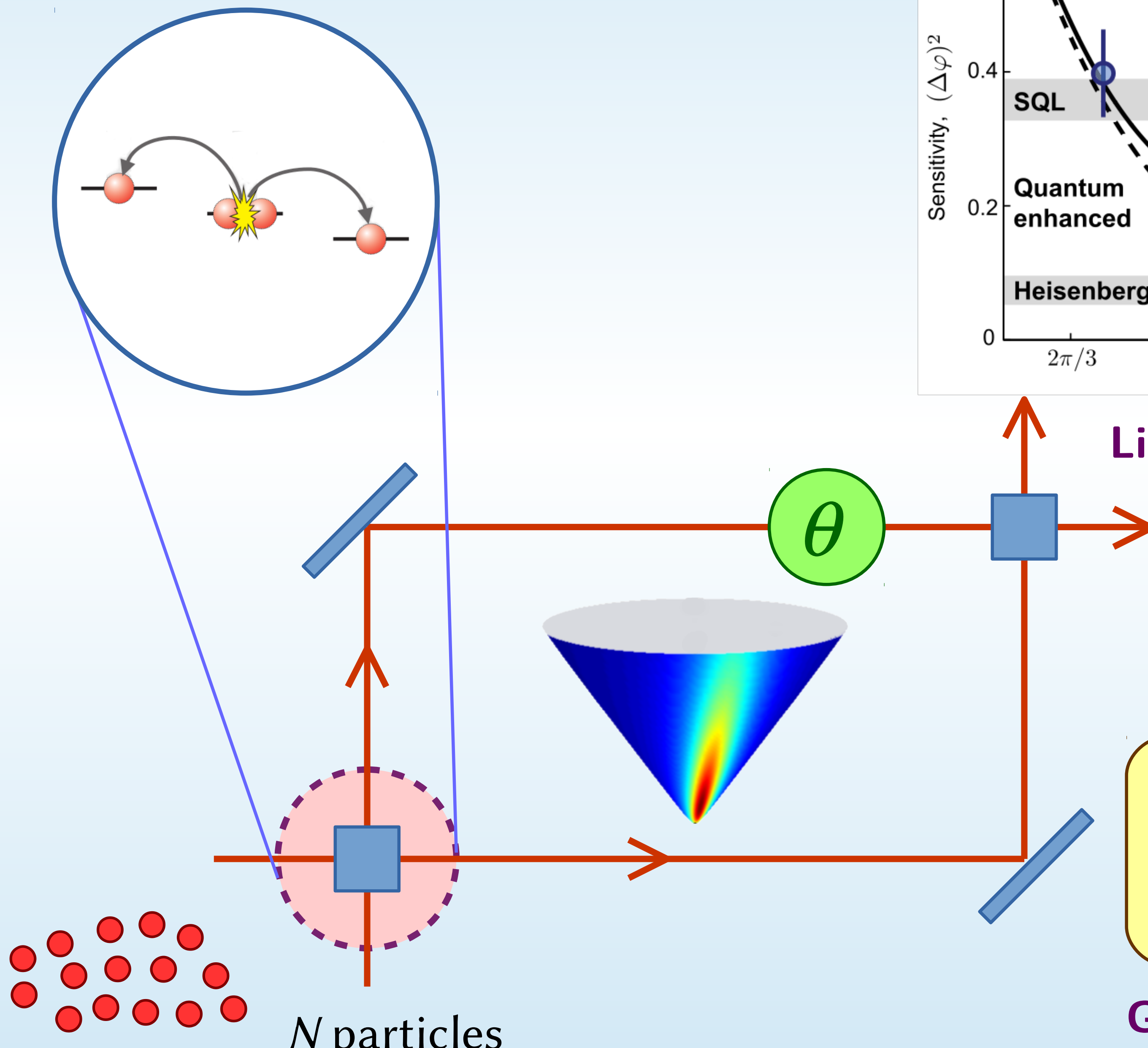
Twin Fock state

Quantum *nonlocal* interferometry

$$\hat{H} = \hat{H}_1\hat{H}_2 + \hat{H}_1\hat{H}_3 + \ldots$$



Linneman *et al*, PRL (2016)



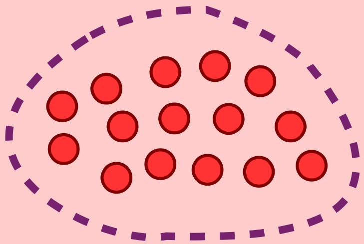
Sub shot noise at T = 0 ✓

Gabbrielli *et al*, PRL (2015)

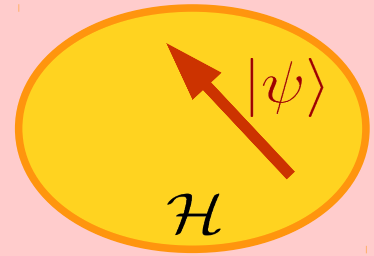
Conclusion

QUANTUM FISHER INFORMATION

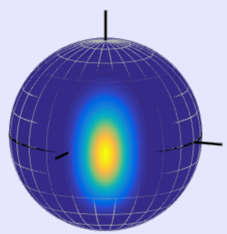
Entanglement witness



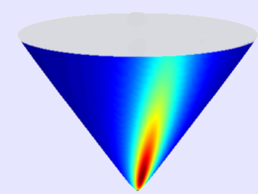
Distinguishability



Local interferometry



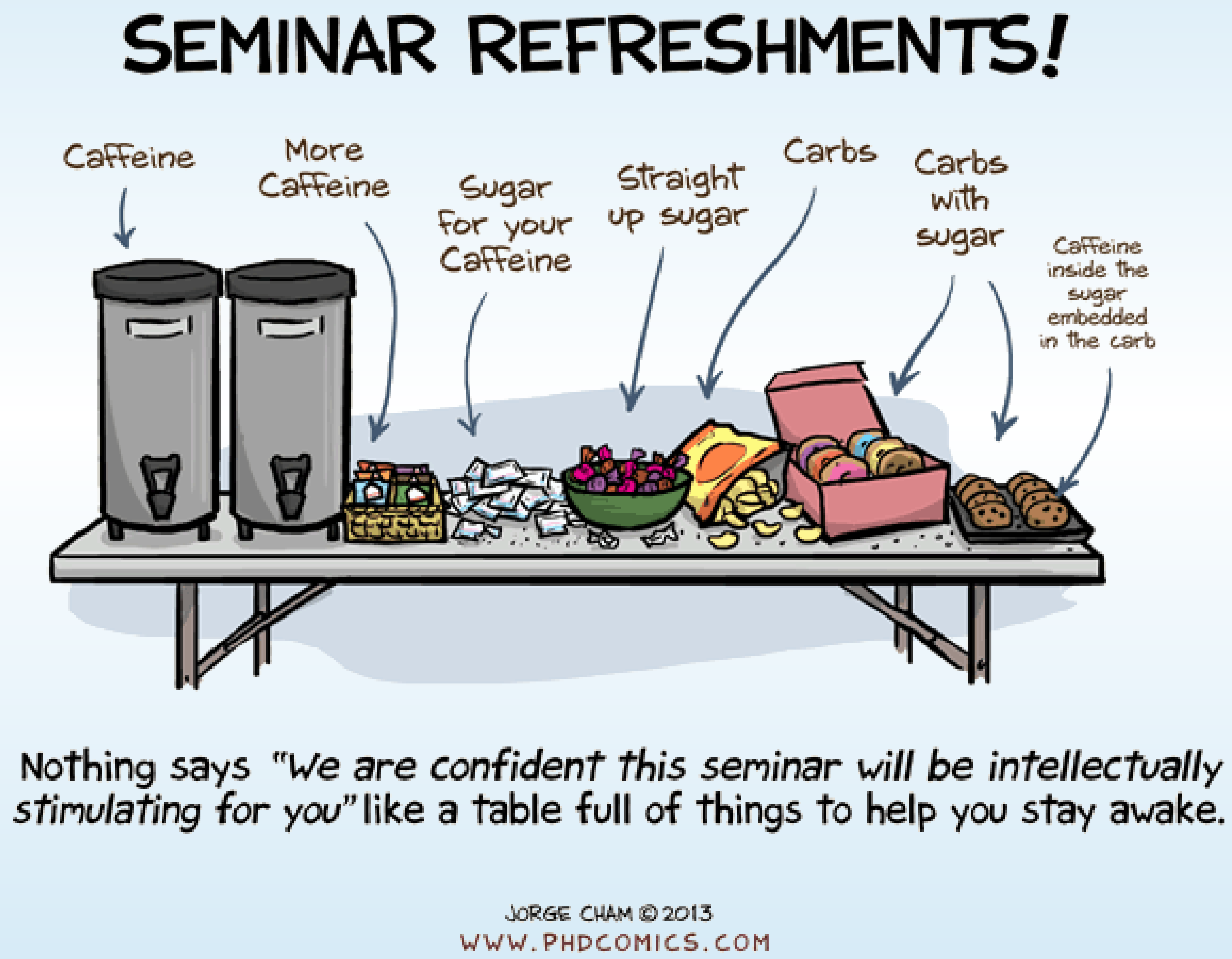
Nonlocal interferometry



Quantum phase transitions

Simulation of
quantum systems

Thank you !


SEMINAR REFRESHMENTS!
Caffeine
More Caffeine
Sugar for your Caffeine
Straight up sugar
Carbs
Carbs with sugar
Caffeine inside the sugar embedded in the carb
Nothing says "We are confident this seminar will be intellectually stimulating for you" like a table full of things to help you stay awake.
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